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Vorwort

Das Tätigkeitsfeld des Fraunhofer-Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe sollen sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen werden.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

Dieter Prätzel-Wolters

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Single-allocation hub location problems with capacity choices

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Abstract

In this paper, an extension to the classical capacitated single-allocation hub location problem is studied in which the size of the hubs is part of the decision making process. For each potential hub a set of capacities is assumed to be available among which one can be chosen. Several formulations are proposed for the problem, which are compared in terms of the bound provided by the linear programming relaxation. Different sets of inequalities are proposed to enhance the models. Several preprocessing tests are also presented with the goal of reducing the size of the models for each particular instance. The results of the computational experiments performed using the proposed models are reported.

Keywords: Hub Location, Capacity decisions, MILP formulations.

1 Introduction

A core task of any traffic network is to provide cost effective means to establish the flow from a set of sources to a set of destinations. Denote by $G = (N, A)$ a complete graph where N is the set of nodes, A is the set of edges and let $n = |N|$. Assume that a flow w_{ij} should be sent from each node i to each node j $(i, j \in N)$. One possibility is to send these flows directly between the corresponding pairs of nodes. However, in practice this is neither efficient nor costly effective because it would imply that a link should be established between each pair of nodes. An alternative is to select some nodes to become hubs and use them as consolidation and redistribution points that together process more efficiently the flow in the network. Accordingly, hubs are nodes in the graph that receive traffic (mail, phone calls, passengers, etc) from different origins (nodes) and redirect this traffic directly to the destination nodes (when a link exists) or to other hubs. The concentration of traffic in the

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hubs and its shipment to other hubs lead to a natural decrease in the overall cost due to economies of scale.

The problem of deciding which nodes should become hubs and how the flow in the network should be consolidated and redistributed defines the basic setting of a hub location problem (Campbell et al. [4]).

The literature on hub location problems has largely increased in the last years as can be observed in the recent survey paper by Alumur and Kara [2]. Despite the significant number of variants of this problem that has been studied, two main classes can be distinguished: single-allocation and multiple-allocation. In the first case, each node that is not a hub is assigned to a single hub. In the latter case, a non-hub node can be assigned to more than one hub.

It is often assumed that the inter-hub network should be a clique, which means that each hub can send flow directly to every other hub. Nevertheless, some papers can be found in the literature in which this structure is relaxed (e.g. Alumur and Kara [1] and Nickel et al. [18]). Another common assumption in the literature is that there are no direct links between non-hub nodes. Therefore, all traffic should be routed via at least one hub.

The hub location problems can be classified as capacitated or uncapacitated depending on whether or not a limit exists on the amount of flow that can be processed in each hub. When this limit exists, it may vary from hub to hub and it often refers to the incoming flow. In fact, often, hubs have to be sized to process incoming information. In many situations, the outgoing traffic does not pose a difficulty in terms of capacity because it does not require any processing.

As its name indicates, the capacitated single-allocation hub location problem (CSAHLP) is a single-allocation hub location problem in which hubs have capacity limits.

Campbell [3] presents the first mixed-integer linear programming (MILP) formulation for the CSAHLP. Ernst and Krishnamoorthy [13] extend the formulation proposed by Skorin-Kapov et al. [20] for an uncapacitated version of the problem to the capacitated case and also propose a new MILP formulation, which is an adaptation to the CSAHLP of a formulation that the same authors proposed for the uncapacitated p-hub median problem (Ernst and Krishnamoorthy [11, 12]). In Ernst and Krishnamoorthy [13] a solution approach based on Simulated Annealing is proposed. The bounds obtained are embedded in a branch-and-bound procedure devised for solving the problem optimally. Labbé et al. [16] study a CSAHLP where there is a capacity on the flow that traverses each hub. A branch-and-cut algorithm is proposed for this problem. Costa et al. [10] present a bi-objective approach. The model proposed by Ernst and Krishnamoorthy [13] is enlarged with the addition of a second objective function to be minimized that quantifies the time to process the flow entering the hubs. More recently, Contreras et al. [6] present a Lagrangean Relaxation enhanced with reduction tests that allows the computation of tight upper and lower bounds for a large set of instances.

In all the above mentioned works, the capacities of the potential hubs are an exogenous decision. No consideration is made about the reasoning behind these capacities. Nevertheless, often, hubs are large structural facilities requiring several strategic decisions to be made in addition to the location decisions. One decision that can hardly be discarded is the dimension/capacity that each hub should have. In this work, this (from an application point of view necessary) extension to the problem is proposed. It will be named the capacitated single-assignment hub location problem with multiple capacity levels (CSAHLPM). It is assumed that there is a set of different sizes available for each potential hub. Accordingly, not only have the hub nodes to be chosen but also the capacity level at which each of them will operate. Each capacity level determines a specific incoming capacity and incurs a specific fixed set-up cost. Economies of scale are assumed for these costs.

In order to illustrate the flexibility and advantages that can be obtained by considering the extension just proposed, consider a problem with 9 nodes as depicted in figure 1. Assume that each square in the grid has a unitary side. Additionally assume that i) each node should send 2 units of flow/traffic to every other node (each node originates 16 units of flow); ii) A hub can be installed in every node with a set-up cost equal to 100 for nodes 2, 5 and 7 and equal to 500 for the other nodes; iii) the flow consolidation capacity of each potential hub is equal to 50 (maximum flow/traffic that a hub can receive from the nodes connected to it); iv) the cost for sending a unit of flow between two hubs is equal to 0.75 times the distance between the hubs; v) the cost for sending one unit of flow between a non-hub node and a hub as well as between a hub and a non-hub is equal to the distance between the nodes involved. vi) the distance between two nodes in the network is given by the euclidean distance.

In the situation just described, taking into account the consolidation capacity of the hubs (50 units of flow) and taking into account that each node originates 16 units of flow, it is clear that at least 3 hubs should be installed. The network depicted in figure 2a represents an optimal solution to the problem. In this figure, the hub nodes are represented by squares. The cost of this solution is $492 + 54(1 + \sqrt{2}) \approx 492 + 54(1 + 1.414) \approx 622.4$, which is obtained taking into account that the total cost of the flow/traffic between non-hubs and hubs is equal to 192, the total cost of the flow between hubs is equal to $54(1 + \sqrt{2})$, and the total set-up cost is equal to 300.

Assume now that it is possible to decide the capacity of a hub to be installed in nodes 2 or 5. In particular, assume that one additional capacity level equal to 80 is available in each of these nodes, with a set-up cost of 120. An optimal solution in this situation is represented in figure 2b. Again, the hub nodes are represented by squares. The cost of this solution is equal to $552 + 32\sqrt{2} \approx 552 + 32 \times 1.414 \approx 597.3$, which is obtained taking into account that the total cost of the flow/traffic between non-hubs and hubs is equal to $192 + 32\sqrt{2}$, the total cost of the flow between hubs is equal to 120, and the total set-up cost is equal to 240. This means that a decrease in the total cost could be achieved by taking advantage of the capacity choices for nodes 2 and 5.

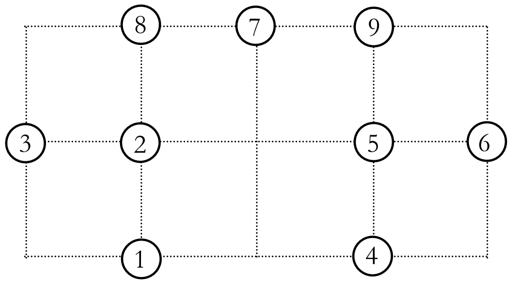
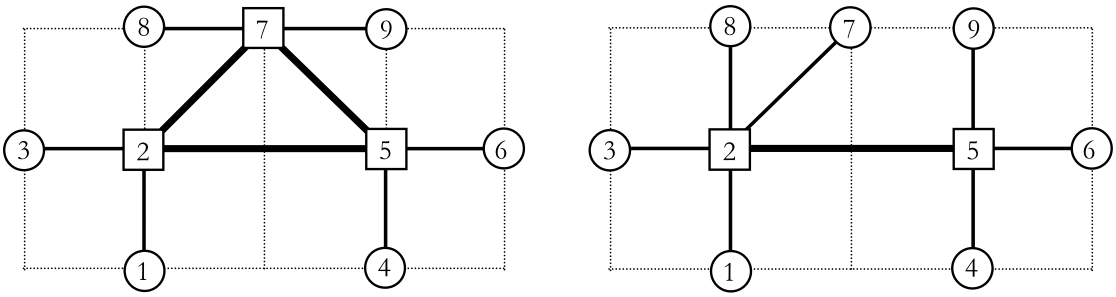


Figure 1: A set of nodes defining a CSAHLP.



(a) (b)

Figure 2: Flexibility in the network design arising from the existence of different capacity levels.

To the best knowledge of the authors, the CSAHLPM has not been treated in the literature. Nevertheless, papers can be found in the literature dealing with the capacitated facility location problems where the capacities of the facilities must be chosen among a set of different possibilities. This is the case in the papers by Correia and Captivo [7], [8], Holmberg [14], and Yoo and Tcha [21].

The remainder of the paper is organized as follows. In the next section, three formulations are proposed for the problem, which result from the extension of existing formulations for the CSAHLP to the new problem. A comparison is made between these models in terms of the bound provided by their linear relaxation. In section 3, a new set of formulations is introduced and analyzed. In section 4, several sets of valid inequalities are proposed in order to enhance the models proposed in sections 2 and 3. In section 5, several preprocessing tests are proposed aiming at reducing the size of the models considered. The computational experiments performed in order to evaluate the models are reported in section 6. The paper ends with some conclusions drawn from the work done.

2 Formulations for the CSAHLPM

Before formulating the problem, the following notation is introduced.

$N = \{1, ..., n\}$	Set of nodes.
$Q_k = \{1, ..., s_k\}$	Set of different capacity levels available for a potential hub to be installed at node k ($k \in N$).
w_{ij}	Flow to be sent from node i to node j ($i, j \in N$).
d_{ij}	Distance between nodes i and j ($i, j \in N$).
α	Cost per unit of flow and per unit of distance between hubs. This value is usually known as discount factor or transfer cost and it is often assumed that $0 \leq \alpha < 1$.
χ	Cost per unit of flow and per unit of distance between a non-hub node and a hub. This value is usually known as collection cost.
δ	Cost per unit of flow and per unit of distance between a hub and a non-hub node. This cost is usually known as distribution cost.
c_{ijkl}	Total cost for sending one unit of flow from node i to node j through hubs k and l. This means that the flow follows the path $i \rightarrow k \rightarrow l \rightarrow j$ and $c_{ijkl} = \chi d_{ik} + \alpha d_{kl} + \delta d_{lj}$ ($i, j, k, l \in N$).
f_k^q	Fixed cost for installing a hub with capacity of level q at node k ($k \in N$, $q \in Q_k$).
Γ_k^q	Capacity of a hub installed at node k with a level of capacity q ($k \in N$, $q \in Q_k$).
$O_i = \sum_{j \in N} w_{ij}$	Total flow originating at node i.
$D_i = \sum_{j \in N} w_{ji}$	Total flow destined for node i.

It is assumed that the distance matrix $[d_{ij}]_{i,j \in N}$ is symmetric and also that $d_{ii} = 0$ ($i \in N$). Moreover, it is assumed that the distances satisfy the triangular inequality. Regarding the flow matrix $[w_{ij}]_{i,j \in N}$, it should be noted that it does not necessarily have a null diagonal.

Costs f_k^q ($k \in N$, $q \in Q_k$) can also include fixed operation costs for the hubs (dependent on the capacity level) when they exist.

Regarding the capacities, for each $k \in N$, the following relation is assumed: $\Gamma_k^{q_1} < \Gamma_k^{q_2}$ for $q_1, q_2 \in Q_k$ such that $q_1 < q_2$.

Clearly, a necessary condition for the feasibility of an instance of the problem is that $\sum_{k\in N}\Gamma_k^{s_k} \geq \sum_{k\in N} O_k$.

In the remainder of the paper $\mathcal{V}(P)$ represents the optimal value of problem P and $\overline{P}$ its linear programming relaxation.

2.1 Mixed-Integer Linear Programming Formulations

A well-known formulation for the CSAHLP was proposed by Campbell [3]. In this formulation, the following decision variables are considered:

$y_{ijkl} =$ Fraction of the flow originated at i destined to j that is routed via hubs k and l by this order $(i, j, k, l \in N)$.

$$x_{ik} = \begin{cases} 1 & \text{if node } i \text{ is assigned to hub } k \\ 0 & \text{otherwise} \end{cases} \quad (i, k \in N).$$

$x_{kk} = 1$ $(k \in N)$ indicates that node k is a hub.

In order to formulate the CSAHLPM, the following decision variables are proposed in addition to the previous ones:

$$z_k^q = \begin{cases} 1 & \text{if node } k \text{ receives a hub with capacity level } q \\ 0 & \text{otherwise} \end{cases} \quad (k \in N,\ q \in Q_k).$$

Accordingly, a MILP formulation for CSAHLPM is the following:

$$(A_C) \quad min \quad \sum_{i\in N}\sum_{j\in N}\sum_{k\in N}\sum_{l\in N} w_{ij}c_{ijkl}y_{ijkl} + \sum_{k\in N}\sum_{q\in Q_k} f_k^q z_k^q \tag{1}$$

$$s.\,to: \quad \sum_{k\in N}\sum_{l\in N} y_{ijkl} = 1 \qquad i, j \in N \tag{2}$$

$$\sum_{j\in N}\sum_{l\in N} (w_{ij}y_{ijkl} + w_{ji}y_{jilk}) = (O_i + D_i)\, x_{ik} \qquad i, k \in N \tag{3}$$

$$\sum_{i\in N} O_i x_{ik} \leq \sum_{q\in Q_k} \Gamma_k^q z_k^q \qquad k \in N \tag{4}$$

$$x_{ik} \leq x_{kk} \qquad i, k \in N \tag{5}$$

$$\sum_{q\in Q_k} z_k^q \leq 1 \qquad k \in N \tag{6}$$

$$x_{ik} \in \{0, 1\} \qquad i, k \in N \tag{7}$$

$$y_{ijkl} \geq 0 \qquad i, j, k, l \in N \tag{8}$$

$$z_k^q \in \{0, 1\} \qquad k \in N,\ q \in Q_k \tag{9}$$

The objective function (1) evaluates the overall cost which is divided into the cost of collection, transfer, distribution, and the cost for installing the hubs. Constraints (2) assure that all the flow is delivered. Constraints (3) impose that if a node is assigned to some hub then all the flow which is originated or destined to the former should go through the latter. Constraints (4) are capacity constraints for the incoming flow on the hubs. Constraints (5) assure that a node can only be assigned to an existing hub. (6) are consistency constraints assuring that for each potential hub at most one capacity level can be chosen. Finally (7), (8) and (9) are domain constraints.

Remark 1 *Note that there is always an optimal solution to A_C such that $y_{ijkl} \in \{0,1\}$, $i,j,k,l \in N$. In fact, for some $i,j,k,l \in N$ if node i is assigned to hub k and node j is assigned to hub l the only possibility of having a fractional y_{ijkl} is to have the flow from i directed via k to several paths leading to l. However, the solution in which this flow is all sent directly to l is still feasible and does not increase the objective value.*

Skorin-Kapov et al. [20] study the uncapacitated p-hub location problem and in particular a formulation that also contains constraints (3). They note that these constraints are very weak and proposed their replacement by the following pair of equalities:

$$\sum_{l \in N} y_{ijkl} = x_{ik} \qquad i,j,k \in N \tag{10}$$

$$\sum_{k \in N} y_{ijkl} = x_{jl} \qquad i,j,l \in N \tag{11}$$

The same can be done for the CSAHLPM (as it is done for the CSAHLP by Ernst and Krishnamoorthy [13]). The new formulation, that is the formulation defined by (1), (2), (10), (11), (4), (5), (6) (7), (8) and (9) will be denoted by A_{SK}.

Another well-known formulation for CSAHLP is proposed by Ernst and Krishnamoorthy [13]. It was obtained from existing formulations for the p-Hub Location Problem. This is a multi-commodity flow formulation in which each commodity refers to the flow originated in a node. The x-variables considered in formulations A_C and A_{SK} are still used. However, now, the following decision variables are considered:

$y^i_{kl} =$ Amount of flow with origin at i that goes through hubs k and l $(i,k,l \in N)$.

In order to formulate the CSAHLPM, the z-variables introduced above are also considered.

Accordingly, a third MILP formulation for the problem is obtained:

$$(A_{EK}) \quad min \quad \sum_{i\in N}\sum_{k\in N} d_{ik}\left(\chi O_i + \delta D_i\right) x_{ik} + \sum_{i\in N}\sum_{k\in N}\sum_{l\in N} \alpha d_{kl} y^i_{kl} + \sum_{k\in N}\sum_{q\in Q_k} f^q_k z^q_k \tag{12}$$

$$s.\,to: \quad (4),\ (5),\ (6),\ (7), (9)$$

$$\sum_{k\in N} x_{ik} = 1 \qquad i \in N \tag{13}$$

$$\sum_{l\in N} y^i_{kl} - \sum_{l\in N} y^i_{lk} = O_i x_{ik} - \sum_{j\in N} w_{ij} x_{jk} \qquad i,k \in N \tag{14}$$

$$\sum_{l\in N} y^i_{kl} \le O_i x_{ik} \qquad i,k \in N \tag{15}$$

$$y^i_{kl} \ge 0 \qquad i,k,l \in N \tag{16}$$

Again, the objective function (12) minimizes the costs for collection, transfer, distribution, and the costs of establishing the hubs. Constraints (13) assure that all nodes are hubs or are assigned to a single hub. Flow conservation is assured by constraints (14), which are, in fact, divergence equations for commodity i at node k. Constraints (15) assure that a y^i_{kl} can only be different from 0 if x_{ik} is equal to one and in this case, all the flow originated in node i is sent to hub k. Finally, (16) are domain constraints.

It should be emphasized that constraints (15) are not part of the model proposed by Ernst and Krishnamoorthy [13]. However, as it is pointed out by Correia et al. [9] these constraints are essential to assure that the model fully describes the set of feasible solutions to the problem.

In table 1 the dimensions of the three models proposed above can be observed, namely in terms of the number of variables and number of constraints. In this table, $s = \max_{i\in N}\{s_i\}$. As in the classical CSAHLP, model A_{SK} is the largest and A_C is the smallest which, as it will be seen in the next section, has a reflection in terms of the linear relaxation bounds produced.

Model	Number of variables		Number of constraints
	Binary	continuous	
A_C	n^4	$O(n^2 + n \times s)$	$3n^2 + 2n$
A_{SK}	n^4	$O(n^2 + n \times s)$	$2n^3 + 2n^2 + 2n$
A_{EK}	n^3	$O(n^2 + n \times s)$	$3n^2 + 3n$

Table 1: Number of constraints and variables in models A_C, A_{SK} and A_{EK}.

It is important to emphasize that when $|Q_k| = 1$, $k \in N$, the CSAHLPM reduces to the CSAHLP and formulations A_C, A_{SK} and A_{EK} reduce to the classical formulations existing for the latter problem.

2.2 A comparison between the different models

To the best knowledge of the authors, the linear relaxation bounds provided by the models existing in the literature for the CSAHLP have only been compared empirically via a set of computational experiments. In this section a theoretical comparison for the models presented above for the CSAHLPM is presented. Note that due to the fact that CSAHLPM reduces to the classical problem when $|Q_k| = 1$ $(k \in N)$, the results presented in this section are also valid for the classical formulations.

Result 1 $\mathcal{V}(\overline{A_{SK}}) \geq \mathcal{V}(\overline{A_{EK}})$.

Proof: Let $S_1 = \{x_{ik}, z_k^q, y_{ijkl}\}$ be a feasible solution for $\overline{A_{SK}}$. Consider the solution $S_2 = \{X_{ik}, Z_k^q, Y_{kl}^i\}$ defined as follows:

For $i, k \in N$, set $X_{ik} = x_{ik}$.

For $k \in N$, $q \in Q_k$, set $Z_k^q = z_k^q$.

For $i, k, l \in N$, set $Y_{kl}^i = \sum_{j \in N} w_{ij} y_{ijkl}$. In what follows, it will be shown that S_2 is a feasible solution to $\overline{A_{EK}}$ and its cost is equal to the cost of S_1.

Constraints (4), (5), (6), (7), (9) and (16) are trivially satisfied. Regarding constraints (13), the following equalities hold (the second and the third equalities are due to constraints (10) and (2), respectively).

For $i \in N$, $\sum_{k \in N} X_{ik} = \sum_{k \in N} x_{ik} = \sum_{k \in N} \sum_{l \in N} y_{ijkl} = 1$.

Proving the feasibility of constraints (14) follows almost directly from constraints (10) and (11). In fact, considering $i, k \in N$ one has:

$\sum_{l \in N} Y_{kl}^i - \sum_{l \in N} Y_{lk}^i - O_i X_{ik} + \sum_{j \in N} w_{ij} X_{jk} =$

$\sum_{l \in N} \sum_{j \in N} w_{ij} y_{ijkl} - \sum_{l \in N} \sum_{j \in N} w_{ij} y_{ijlk} - \left(\sum_{j \in N} w_{ij}\right) X_{ik} + \sum_{j \in N} w_{ij} X_{jk} =$

$\sum_{l \in N} \sum_{j \in N} w_{ij} y_{ijkl} - \sum_{l \in N} \sum_{j \in N} w_{ij} y_{ijlk} - \sum_{j \in N} w_{ij} X_{ik} + \sum_{j \in N} w_{ij} X_{jk} =$

$\sum_{l \in N} \sum_{j \in N} w_{ij} y_{ijkl} - \sum_{j \in N} \sum_{l \in N} w_{ij} y_{ijlk} - \sum_{j \in N} \left(w_{ij} \sum_{l \in N} y_{ijkl}\right) + \sum_{j \in N} w_{ij} X_{jk} =$

$\sum_{l \in N} \sum_{j \in N} w_{ij} y_{ijkl} - \sum_{j \in N} \left(w_{ij} \sum_{l \in N} y_{ijlk}\right) - \sum_{j \in N} \sum_{l \in N} w_{ij} y_{ijkl} + \sum_{j \in N} w_{ij} X_{jk} =$

$- \sum_{j \in N} \left(w_{ij} \sum_{l \in N} y_{ijlk}\right) + \sum_{j \in N} w_{ij} X_{jk} =$

$- \sum_{j \in N} w_{ij} X_{jk} + \sum_{j \in N} w_{ij} X_{jk} = 0.$

Regarding the objective function one has:

$\sum_{i \in N} \sum_{k \in N} d_{ik} (\chi O_i + \delta D_i) X_{ik} + \sum_{i \in N} \sum_{k \in N} \sum_{l \in N} \alpha d_{kl} Y_{kl}^i =$

$\sum_{i \in N} \sum_{k \in N} d_{ik} (\chi O_i + \delta D_i) x_{ik} + \sum_{i \in N} \sum_{k \in N} \sum_{l \in N} \sum_{j \in N} \alpha d_{kl} w_{ij} y_{ijkl} =$

$\sum_{i\in N}\sum_{k\in N}\sum_{j\in N}\sum_{l\in N} w_{ij}\chi d_{ik}y_{ijkl} + \sum_{i\in N}\sum_{k\in N}\sum_{j\in N}\sum_{l\in N} w_{ji}\delta d_{ik}y_{jilk}+$

$\sum_{i\in N}\sum_{k\in N}\sum_{l\in N}\sum_{j\in N} \alpha d_{kl}w_{ij}y_{ijkl} =$

$\sum_{i\in N}\sum_{j\in N}\sum_{k\in N}\sum_{l\in N} w_{ij}\left(\chi d_{ik} + \alpha d_{kl} + \delta d_{lj}\right) y_{ijkl}.$

We have proved that the two solutions have the same cost and hence $\mathcal{V}(\overline{A_{SK}}) \geq \mathcal{V}(\overline{A_{EK}})$.

$\square$

The next example shows that the inequality established in the previous result may be strict.

Example 1 *Consider an instance of the CSAHLPM where* $N = \{1,2,3\}$, $s_k = 2$ $(k \in N)$, $\chi = \delta = 1$ *and* $\alpha = 0.7$. *Additionally, consider*

$$W = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \\ 2 & 2 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 21 & 24 \\ 13 & 15 \\ 28 & 33 \end{bmatrix}, \quad f = \begin{bmatrix} 25 & 26 \\ 48 & 50 \\ 42 & 44 \end{bmatrix}$$

For this instance, $\mathcal{V}(\overline{A_{EK}}) = 40.275 < \mathcal{V}(\overline{A_{SK}}) = 41.6 < \mathcal{V}(A_{EK}) = \mathcal{V}(A_{SK}) = 52.$
$\triangle$

Result 2 $\mathcal{V}(\overline{A_{SK}}) \geq \mathcal{V}(\overline{A_C})$.

Proof: As the two models have the same objective function it suffices to prove that the set of feasible solutions for model $\overline{A_C}$ contains the set of feasible solutions for model $\overline{A_{SK}}$.

Considering a feasible solution $S = \{x_{ik}, z_k^q, y_{ijkl}\}$ for $\overline{A_{SK}}$, it is only necessary to prove that the set of constraints (3) is satisfied by S.

As S is feasible for $\overline{A_{SK}}$, due to constraints (10) and (11) one has:

$\sum_{l\in N} y_{ijkl} = x_{ik} \qquad i,j,k \in N \qquad (i)$

$\sum_{l\in N} y_{jilk} = x_{ik} \qquad i,j,k \in N \qquad (ii)$

Multiplying each equality (i) by w_{ij} and adding over j one obtains:

$\sum_{l\in N}\sum_{j\in N} w_{ij}y_{ijkl} = \sum_{j\in N} w_{ij}x_{ik} \qquad i,k \in N \qquad (iii)$

The multiplication of each equality (ii) by w_{ji} and the addition over j leads to the next equalities.

$\sum_{l\in N}\sum_{j\in N} w_{ji}y_{jilk} = \sum_{j\in N} w_{ji}x_{ik} \qquad i,k \in N \qquad (iv)$

The summation of (iii) and (iv) leads to constraints (3).

$\square$

Another illustrative example demonstrates that the inequality stated in the previous result can be strict.

Example 2 *Consider an instance of the CSAHLPM where* $N = \{1, 2\}$*,* $s_k = 2$ $(k \in N)$*,* $\chi = \delta = 1$ *and* $\alpha = 0.9$*. Additionally, consider*

$$W = \begin{bmatrix} 10 & 20 \\ 20 & 70 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 60 & 240 \\ 50 & 250 \end{bmatrix}, \quad f = \begin{bmatrix} 40 & 120 \\ 50 & 200 \end{bmatrix}$$

For this instance, $\mathcal{V}(\overline{A_C}) = 121 < \mathcal{V}(\overline{A_{SK}}) = 123 < \mathcal{V}(A_C) = \mathcal{V}(A_{SK}) = 260$.
$\triangle$

The two previous results establish that formulation A_{SK} dominates formulations A_C and A_{SK} in terms of the bound provided by the linear relaxation. Moreover, the examples above show that this dominance may be strict. Note that this fact had already been established empirically for the corresponding models existing for the CSAHLP. Despite having found the best model in terms of the linear relaxation bound among the three models presented for the CSAHLPM, it might be interesting to compare theoretically the linear relaxation bounds provided by models A_C and A_{EK} so that a full ranking could possibly be established between the three models. The following conjecture is empirically supported.

Conjecture 1 $\mathcal{V}(\overline{A_{EK}}) \geq \mathcal{V}(\overline{A_C})$.

So far, this conjecture has not been proved or disproved. The relation between both formulations is still an open question. Nevertheless, the following example illustrates that for some instances, formulation A_{EK} can be strictly better than formulation A_C in terms of the linear relaxation bound.

Example 3 *Consider again the instance presented in example 2. For this instance,*

$$\mathcal{V}(\overline{A_C}) = 121 < \mathcal{V}(\overline{A_{EK}}) = 123 < \mathcal{V}(A_C) = \mathcal{V}(A_{EK}) = 260.$$

$\triangle$

3 New formulations for the CSAHLPM

In this section another set of formulations for the CSAHLPM is presented that is motivated by the fact that for each hub at most one capacity level should be chosen. In addition to the y-variables proposed by Ernst and Krishnamoorthy [13] the following decision variables are introduced:

$$t_{ik}^q = \begin{cases} 1 & \text{if node } i \text{ is assigned to hub } k \text{ which has capacity level } q \\ 0 & \text{otherwise} \end{cases} \qquad (i, k \in N,\ q \in Q_k).$$

$t_{kk}^q = 1$ ($k \in N$) indicates that node k is a hub with capacity at level q.

The relation between the former variables x_{ik} and z_k^q and the new variables is straightforward:

$$x_{ik} = \sum_{q \in Q_k} t_{ik}^q \qquad i, k \in N. \tag{17}$$

and

$$z_k^q = t_{kk}^q \qquad k \in N,\ q \in Q_k. \tag{18}$$

The use of the transformation defined by relations (17) and (18) together with the formulations A_C, A_{EK} and A_{SK} presented in the previous section is the main ingredient for obtaining a new set of formulations for the CSAHLPM. Consider formulation A_{EK} (the reasoning presented next is also valid using the other formulations). By replacing $x-$ and $z-$variables in formulation A_{EK} according to relations (17) and (18), the following formulation is obtained:

$$(TA_{EK}) \quad min \quad \sum_{i \in N} \sum_{k \in N} \left(d_{ik} \left(\chi O_i + \delta D_i \right) \sum_{q \in Q_k} t_{ik}^q \right) + \sum_{i \in N} \sum_{k \in N} \sum_{l \in N} \alpha d_{kl} y_{kl}^i + \sum_{k \in N} \sum_{q \in Q_k} f_k^q t_{kk}^q \tag{19}$$

$$s.\,to: \quad \sum_{k \in N} \sum_{q \in Q_k} t_{ik}^q = 1 \qquad i \in N \tag{20}$$

$$\sum_{l \in N} y_{kl}^i - \sum_{l \in N} y_{lk}^i = O_i \sum_{q \in Q_k} t_{ik}^q - \sum_{j \in N} \left(w_{ij} \sum_{q \in Q_k} t_{jk}^q \right) \qquad i, k \in N \tag{21}$$

$$\sum_{l \in N} y_{kl}^i \le O_i \sum_{q \in Q_k} t_{ik}^q \qquad i, k \in N \tag{22}$$

$$\sum_{i \in N} \left(O_i \sum_{q \in Q_k} t_{ik}^q \right) \le \sum_{q \in Q_k} \Gamma_k^q t_{kk}^q \qquad k \in N \tag{23}$$

$$\sum_{q \in Q_k} t_{ik}^q \le \sum_{q \in Q_k} t_{kk}^q \qquad i, k \in N \tag{24}$$

$$y_{kl}^i \ge 0 \qquad i, k, l \in N \tag{16}$$

$$t_{ik}^q \in \{0, 1\} \qquad i, k \in N,\ q \in Q_k \tag{25}$$

Note that the transformed constraints (6), (7), and (9) are redundant.

The following result holds:

Result 3 $\mathcal{V}(\overline{TA_{EK}}) \ge \mathcal{V}(\overline{A_{EK}})$.

Proof: The result follows simply by considering a feasible solution to $\overline{TA_{EK}}$, say, $\{y^i_{kl}, t^q_{ik}\}$ and by considering the solution $\{x_{ik}, y^i_{kl}, z^q_k\}$ obtained from the former one by using (17) and (18). It is trivial to conclude that $\{x_{ik}, y^i_{kl}, z^q_k\}$ is feasible to $\overline{A_{EK}}$ and that both solutions give the same value to the objective function in the corresponding model.

□

Although looking promising when considering the previous result, the transformation defined by (17) and (18) is not a valid transformation for A_{EK} in the sense that model TA_{EK} is not a valid model for the CSAHLPM. In fact, the constraints in the latter formulation do not avoid having, for instance, $t^q_{kk} = 1$ with $t^{q'}_{ik} = 1$ for $i \neq k$ and $q' \neq q$, which would make no sense. Nevertheless, this short example makes clear what is needed in model TA_{EK} to produce a valid model for the problem: One only needs to disaggregate constraints (23), which leads to

$$\sum_{i \in N} O_i t^q_{ik} \leq \Gamma^q_k t^q_{kk} \qquad k \in N,\ q \in Q_k \tag{26}$$

Denote by U_{EK} model TA_{EK} with (23) replaced by (26). By construction and by transitivity using result 3 the following result has just been proved:

Result 4 $\mathcal{V}(\overline{U_{EK}}) \geq \mathcal{V}(\overline{A_{EK}})$.

Following the reasoning above, the new variables t^q_{ik} can also be incorporated in the model derived from the one by Campbell [3], leading to a projected model TA_C and, consequently, to:

$$(U_C) \quad min \quad \sum_{i \in N}\sum_{j \in N}\sum_{k \in N}\sum_{l \in N} w_{ij} c_{ijkl} y_{ijkl} + \sum_{k \in N}\sum_{q \in Q_k} f^q_k t^q_{kk} \tag{27}$$

$$s.\,to: \quad \sum_{k \in N}\sum_{l \in N} y_{ijkl} = 1 \qquad i, j \in N \tag{2}$$

$$\sum_{j \in N}\sum_{l \in N} (w_{ij} y_{ijkl} + w_{ji} y_{jikl}) = (O_i + D_i) \sum_{q \in Q_k} t^q_{ik} \qquad i, k \in N \tag{28}$$

$$\sum_{i \in N} O_i t^q_{ik} \leq \Gamma^q_k t^q_{kk} \qquad k \in N,\ q \in Q_k \tag{26}$$

$$\sum_{q \in Q_k} t^q_{ik} \leq \sum_{q \in Q_k} t^q_{kk} \qquad i, k \in N \tag{24}$$

$$\sum_{q \in Q_k} t^q_{kk} \leq 1 \qquad k \in N \tag{29}$$

$$t^q_{ik} \in \{0, 1\} \qquad i, k \in N,\ q \in Q_k \tag{25}$$

$$y_{ijkl} \geq 0 \qquad i, j, k, l \in N \tag{8}$$

Denote by TA_{SK} (U_{SK}) the model arising when in model TA_C (U_C) constraints (28) are replaced by

$$\sum_{l \in N} y_{ijkl} = \sum_{q \in Q_k} t_{ik}^q \qquad i,j,k \in N \tag{30}$$

$$\sum_{k \in N} y_{ijkl} = \sum_{q \in Q_j} t_{jl}^q \qquad i,j,l \in N \tag{31}$$

In table 2 the differences between models U_C, U_{SK} and U_{EK} in terms of the number of variables and number of constraints can be observed. As in section 2, $s = \max_{i \in N}\{s_i\}$.

Model	Number of variables		Number of constraints
	Binary	continuous	
U_C	n^4	$O(n^2 \times s)$	$O(3n^2 + n + n \times s)$
U_{SK}	n^4	$O(n^2 \times s)$	$O(2n^3 + 2n^2 + n + n \times s)$
U_{EK}	n^3	$O(n^2 \times s)$	$O(2n^2 + 2n + n \times s)$

Table 2: Number of constraints and variables in models U_C, U_{SK} and U_{EK}.

Following a reasoning similar to the one that led to result 4, the following result can be easily proved:

Result 5

1 $\mathcal{V}(\overline{U_C}) \geq \mathcal{V}(\overline{A_C})$.

2 $\mathcal{V}(\overline{U_{SK}}) \geq \mathcal{V}(\overline{A_{SK}})$.

Remark 2 *A clear advantage of formulations U_{EK}, U_C and U_{SK} is the possibility of easily including additional costs associated with the hubs. For instance, when sizing decisions can be made, it is often the case that they reflect variable processing costs, that is, costs associated with the amount of flow processed in the hub. Such a situation can be easily modeled by considering the additional notation p_{kq} denoting the unitary processing cost for a hub with capacity level q operating at k ($k \in N$, $q \in Q_k$) and by adding the following expression to the objective function of models U_{EK}, U_C and U_{SK}:*

$$\sum_{i \in N} \left(O_i \sum_{k \in N} \sum_{q \in Q_k} p_{kq} t_{ik}^q \right)$$

Next, a small example is presented showing that the inequalities stated in results 4 and 5 can be strict.

Example 4 *Consider again the instance presented in example 2 (page 11). For that instance,*

$$\mathcal{V}(\overline{A_{EK}}) = 123 < \mathcal{V}(\overline{U_{EK}}) \simeq 246.67 < \mathcal{V}(U_{EK}) = 260.$$

$$\mathcal{V}(\overline{A_C}) \simeq 121 < \mathcal{V}(\overline{U_C}) \simeq 233.33 < \mathcal{V}(U_C) = 260.$$

$$\mathcal{V}(\overline{A_{SK}}) = 123 < \mathcal{V}(\overline{U_{SK}}) \simeq 246.67 < \mathcal{V}(U_{SK}) = 260.$$

△

Finally, the following result establishes that in terms of the linear relaxation bound, the ranking that was established in section 2 for formulations A is maintained when considering formulations U. Accordingly we have:

Result 6

1 $\mathcal{V}(\overline{U_{SK}}) \geq \mathcal{V}(\overline{U_{EK}})$.

2 $\mathcal{V}(\overline{U_{SK}}) \geq \mathcal{V}(\overline{U_C})$.

Proof: The result follows from the fact that formulations TA_C, TA_{EK} and TA_{SK} result from applying the linear transformation defined by (17) and (18) to formulations A_C, A_{EK} and A_{SK}, respectively, and also by the fact that the disaggregation that leads to models U_C, U_{EK} and U_{SK} (from models TA_C, TA_{EK} and TA_{SK}) is the same in the three cases.

□

In figure 3, a summary of all relations that have been established between the formulations introduced for the CSAHLPM is depicted. In this figure, an arc exists between two formulations when the first dominates the second in terms of the linear relaxation bound. In the root of the resulting oriented network one can find formulation U_{SK}, being the one providing the strongest linear relaxation bound.

4 Valid inequalities

In this section, several sets of valid inequalities are presented aiming at enhancing the models proposed in the previous sections.

$$
\begin{tikzcd}
 & U_{SK} \arrow[dl] \arrow[d] \arrow[dr] & \\
U_C \arrow[d] & A_{SK} \arrow[dl] \arrow[dr] & U_{EK} \arrow[d] \\
A_C & & A_{EK}
\end{tikzcd}
$$

Figure 3: Relation between the different formulations in terms of the linear relaxation bound.

4.1 Enhanced consistency constraints

A first enhancement that can be proposed for formulations A_C, A_{SK} and A_{EK} regards the replacement of consistency constraints (6) by

$$\sum_{q \in Q_k} z_k^q = x_{kk} \qquad k \in N \tag{32}$$

As stated in the following result, this replacement gives also the possibility of removing constraints (5) in models A_C, A_{EK} and A_{SK}.

Result 7 *Constraints* (5) *are redundant in the presence of* (32)

Proof: Consider $k \in N$. Constraints (4), (7), (9), and (32) together allow us to write:

$x_{kk} = 0 \quad \Rightarrow \quad \sum_{q \in Q_k} z_k^q = 0 \quad \Rightarrow \quad z_k^q = 0 \ \forall q \in Q_k \quad \Rightarrow \quad \sum_{q \in Q_k} \Gamma_k^q z_k^q = 0 \quad \Rightarrow \quad \sum_{i \in N} O_i x_{ik} = 0 \quad \Rightarrow \quad x_{ik} = 0 \ \forall i, k \in N$

Conversely, if $x_{ik} = 1$ for some $i, k \in N$ then one has:

$x_{ik} = 1 \quad \Rightarrow \quad \sum_{i \in N} O_i x_{ik} > 0 \quad \Rightarrow \quad \sum_{q \in Q_k} \Gamma_k^q z_k^q > 0 \quad \Rightarrow \quad \sum_{q \in Q_k} z_k^q > 0 \quad \Rightarrow \quad x_{kk} > 0 \quad \Rightarrow \quad x_{kk} = 1$

$\square$

Note that if relations (17) and (18) are used to write constraints (32), the resulting equalities are redundant in the linear relaxations of U_C, U_{SK} and U_{EK}.

4.2 Disaggregation of the projected strong formulation cuts

One possibility for enhancing models U_C, U_{EK} and U_{SK} consists simply in disaggregating constraints (24), which leads to

$$t_{ik}^q \leq t_{kk}^q \qquad i, k \in N,\ q \in Q_k \tag{33}$$

In section 3, it was the disaggregation of constraints (26) that led to a valid reformulation for the CSAHLPM. It is worth noting that valid reformulations would also have been obtained if constraints (24) had been disaggregated instead of (26) that is if (33) had been considered in the place of (24).

4.3 Minimum number of hubs to be opened

One type of inequality that can be used to enhance the models proposed in the previous sections is motivated by discrete capacitated facility location models: A lower bound is stated on the minimum number of facilities to be opened.

In order to evaluate the minimum number of hubs that should be opened in the case of the CSAHLPM, only the maximum capacity available will be considered for each potential hub. Accordingly, consider the set of capacities, $\Gamma_1^{s_1}, ..., \Gamma_n^{s_n}$. Denote by $\Gamma_{k_1}^{s_{k_1}}, ..., \Gamma_{k_n}^{s_{k_n}}$ those capacities ordered non increasingly. Denote by R a lower bound on the minimum number of hubs that must be installed in order to assure the existence of a feasible solution. R can be obtained as the value that satisfies

$$\sum_{l=1}^{R-1} \Gamma_{k_l}^{s_{k_l}} < \sum_{k \in N} O_k \leq \sum_{l=1}^{R} \Gamma_{k_l}^{s_{k_l}} \tag{34}$$

Therefore, the following inequalities can be added to models A_C, A_{SK} and A_{EK}:

$$\sum_{k \in K} x_{kk} \geq R \tag{35}$$

Considering again relations (17), it is possible to write the previous inequalities using decision variables t:

$$\sum_{k \in K} \sum_{q \in Q_k} t_{kk}^q \geq R \tag{36}$$

These inequalities can thus be added to U_{EK}, U_C and U_{SK}.

4.4 Other inequalities

Another set of inequalities can be added to the models by noting that for $i, j, k, l \in N$ variable y_{ijkl} can be equal to 1 only if $x_{ik} = 1$ and $x_{jl} = 1$. Therefore, one can write:

$$2y_{ijkl} \leq x_{ik} + x_{jl} \qquad i, j, k, l \in N \tag{37}$$

These inequalities can be written using t-variables as follows:

$$2y_{ijkl} \leq \sum_{q \in Q_k} t_{ik}^q + \sum_{q \in Q_l} t_{jl}^q \qquad i, j, k, l \in N \tag{38}$$

Conversely, if $x_{ik} = 1$ and $x_{jl} = 1$ then y_{ijkl} should be equal to 1. Accordingly, one has

$$y_{ijkl} \geq x_{ik} + x_{jl} - 1 \qquad i, j, k, l \in N \tag{39}$$

Writing these inequalities using variables t leads to:

$$y_{ijkl} \geq \left(\sum_{q \in Q_k} t_{ik}^q + \sum_{q \in Q_l} t_{jl}^q \right) - 1 \qquad i, j, k, l \in N \tag{40}$$

4.5 Synthesis

All the inequalities presented can be summarized as follows:

Formulations A_C, A_{SK} and A_{EK}	Formulations U_C, U_{SK} and U_{EK}
(32) $\sum_{q \in Q_k} z_k^q = x_{kk}, \quad k \in N$	
	(33) $t_{ik}^q \leq t_{kk}^q, \quad i, k \in N,\ q \in Q_k$
(35) $\sum_{k \in K} x_{kk} \geq R$	(36) $\sum_{k \in K} \sum_{q \in Q_k} t_{kk}^q \geq R$
(37) $2y_{ijkl} \leq x_{ik} + x_{jl}, \quad i, j, k, l \in N$	(38) $2y_{ijkl} \leq \sum_{q \in Q_k} t_{ik}^q + \sum_{q \in Q_l} t_{jl}^q, \quad i, j, k, l \in N$
(39) $y_{ijkl} \geq x_{ik} + x_{jl} - 1, \quad i, j, k, l \in N$	(40) $y_{ijkl} \geq \left(\sum_{q \in Q_k} t_{ik}^q + \sum_{q \in Q_l} t_{jl}^q \right) - 1, \quad i, j, k, l \in N$

5 Preprocessing

In this section, the possibility of performing a preprocessing in the models presented in the previous sections is explored by making use of the data defining a specific instance. The goal is to reduce the size of the formulations by setting values to some variables.

A first set of preprocessing tests that are suited for formulation A_C, A_{EK} and A_{SK}, is presented in table 3.

Test 1 results from the fact that each potential hub should have capacity to process at least the flow originated in it. In the limit, if the largest capacity available for the potential hub is not enough to process the flow originated in the node then it cannot be a hub and, consequently, no other node can be assigned to it.

Test 2, results from the combination of the flow originated in two different nodes with the need to have enough processing capacity when one of the nodes is a hub with the other node assigned to it.

Test 3 explores a condition under which it might be better to open a hub in some node than to assign this node to some other hub. The test results from the fact that under the

	Condition	Variable fixing
Test 1	$O_k > \Gamma_k^q$ for some $k \in N$ and $q \in Q_k \setminus \{s_k\}$	$z_k^q = 0$
	$O_k > \Gamma_k^{s_k}$ for some $k \in N$	$z_k^{s_k} = 0$ $x_{ik} = 0,\ i \in N$
Test 2	$O_i + O_k > \Gamma_k^{s_k}$ for some $i, k \in N$, $i \neq k$	$x_{ik} = 0$
Test 3	$d_{ik}(\chi O_i + \delta D_i) > f_i^{q'} + \alpha d_{ik}(O_i + D_i - 2w_{ii})$ for some $i, k \in N$, $k \neq i$, where $q' = min\{q : q \in Q_i \wedge O_i \leq \Gamma_i^q\}$	$x_{ik} = 0$
Test 4	x_{ik} is set to 0 for some $i, k \in N$	$y_{kl}^i = 0,\ l \in N$ $y_{ijkl} = 0,\ j, l \in N$ $y_{jilk} = 0,\ j, l \in N$

Table 3: Preprocessing tests for formulations A_C, A_{EK} and A_{SK}.

condition presented (and assuming that q' exists), node i will never be assigned to hub k because it will be cheaper to open a hub in i with capacity level q'.

Finally, test 4 is an immediate consequence of the fact that if a node i is not assigned to a hub k then, i can not send flow directly to k.

Apart from test 1, the other tests are extensions to the CSAHLPM of similar tests proposed by Ernst and Krishnamoorthy [13] for the CSAHLP.

Table 4 presents a set of preprocessing tests that can be applied to formulation U and are straightforward adaptations to the formulations of tests 1-4.

	Condition	Variable fixing
Test 5	$O_k > \Gamma_k^q$ for some $k \in N$ and $q \in Q_k$	$t_{ik}^q = 0,\ i \in N$
Test 6	$O_i + O_k > \Gamma_k^q$ for some $i, k \in N$, $i \neq k$, $q \in Q_k$	$t_{ik}^q = 0$
Test 7	$d_{ik}(\chi O_i + \delta D_i) > f_i^{q'} + \alpha d_{ik}(O_i + D_i - 2w_{ii})$ for some $i, k \in N$, where $q' = min\{q : q \in Q_i \wedge O_i \leq \Gamma_i^q\}$	$t_{ik}^q = 0,\ q \in Q_k$
Test 8	For some $i, k \in N$, $t_{ik}^q = 0,\ q \in Q_k$	$y_{kl}^i = 0,\ l \in N$ $y_{ijkl} = 0,\ j, l \in N$ $y_{jilk} = 0,\ j, l \in N$

Table 4: Preprocessing tests for formulations U_C, U_{EK} and U_{SK}.

6 Computational experiments

In this section the computational experiments performed to evaluate the different formulations presented for the CSAHLPM are reported. In subsection 6.1, the data considered in the experiments is presented. In subsection 6.2, the results obtained are presented and discussed.

6.1 Test data

The first set of benchmark instances presented for hub location problems is the well-known CAB data set due to O'Kelly [19]. This set of instances has been widely used in the literature. It is based on airline passenger flow among 25 cities in the United States. For the study presented in the current paper, this set was not considered because neither capacities nor fixed set-up costs for the hubs are included. Instead, the AP data set introduced by Ernst and Krishnamoorthy [11] was taken as a basis to build the test instances for the CSAHLPM. In fact, in this case, capacities and fixed costs are also given.

In the case of the CSAHLPM there are capacity levels for each potential hub. In order to obtain a set of instances for the problem, the same basic data presented by Ernst and Krishnamoorthy [11] (available in the OR Library [17]) was considered. Accordingly, instances with 10, 20, 25, 40 and 50 nodes were considered. Nowadays, instances of the CSAHLP with a much larger number of nodes can be solved optimally (see Contreras [5]). Nevertheless, recall that in the case of the CSAHLPM one additional dimension is considered in the decision making process, which is the capacity choice.

The number of capacity levels available for each hub was set the same for all nodes and four possibilities were considered: 2, 3, 4 and 5. Therefore, 20 possible combinations exist using the number of nodes and the number of capacity levels. For each combination, 4 instances were generated as follows:

- For each $k \in N$, $\Gamma_k^{s_k} = \Gamma_k$ and $\Gamma_k^q = 0.7 \times \Gamma_k^{q+1}$, $q = 1, \ldots, s_k - 1$ where Γ_k denotes the 'tight' capacity for the potential hub k in the corresponding instance in the AP data set. This means that the largest capacity level was set equal to the tight capacity of the corresponding node in the AP data set instance and then recursively, each capacity level was set equal to 70% of the capacity level immediately above. The reasoning behind this procedure was to obtain more challenging instances for the CSAHLPM problem. In fact, a set of preliminary computational tests showed that the problem becomes very easy to solve when the capacities are loose. The procedure just described assures tighter capacities.
- For each $k \in N$, $f_k^{s_k} = f_k$ and $f_k^q = \rho \times \Gamma_k^q \times \frac{f_k^{q+1}}{\Gamma_k^{q+1}}$, $q = 1, \ldots, s_k - 1$ where f_k denotes

the set-up cost of the potential hub k in the instance retrieved from the AP data set. This means that the set-up cost of a hub at its highest capacity level was set equal to the set-up cost of the same potential hub in the AP data instance. Then, an increase defined by the factor ρ was assumed for the unitary capacity cost when the capacity level decreased. Two values were considered for factor ρ: 1.1 and 1.2, which define two different economies of scale for the set-up costs. In the first case, a 10% increase is considered for the unitary capacity cost when the capacity level decreases. In the second case, this increase is 20%. Therefore a soft and a strong economy of scale are being considered. Due to the fact that there are two types of costs in the instances from the AP data set ('loose' and 'tight'), two types of costs were also considered for the CSAHLPM by considering f_k 'loose' or 'tight'.

6.2 Computational results

In order to evaluate the formulations presented in sections 2 and 3, the general solver CPLEX 11.0 [15] was used. No change was made in the default values of the solver parameters apart from the time limit which was set to 2 hours. The tests were run on a machine with an INTEL processor with 2.9 GHz and 2 GB of RAM.

A set of preliminary computations showed a clear superiority of formulations A_{EK} and U_{EK} when compared with formulations A_C, U_C, A_{SK} and U_{SK}. In particular, for the instances with 40 and 50 nodes it was often the case that when solving the linear relaxation of formulations A_C, A_{SK}, U_C and U_{SK} an out-of-memory error occurred. Finding the optimal solution to the problem was often a burden even for small instances. Therefore, in order to narrow the search for a good formulation for the CSAHLPM, the results obtained with formulations A_{EK} and U_{EK} are the only ones reported below. These results are presented in tables 5-8. Each of these tables reports the results for the 20 instances associated with a specific type of costs (tight or loose) and associated with a specific economy of scale in the set-up costs for the hubs (1.1 or 1.2).

In the first column of tables 5-8 the models considered are presented. For instance '$P-(A)+(B)$' denotes formulation P removing constraints (A) and adding constraints (B). 'prep' means that the preprocessing tests were applied. The second and third columns in tables 5-8 refer to the linear programming relaxation namely the average gap (%) and the average CPU time (seconds) for the 20 instances associated with the table. In columns 4-7 the results associated with the optimal solution are reported. In column 4, the number of instances (out of 20) in which the time limit was attained or an out-of-memory error occurred is presented. For these instances, the average gap (%) of the best feasible solution is depicted in column 5. Columns 6 and 7 present average values for the successful instances (neither the time limit is achieved nor an out-of-memory error occurs) namely the CPU time and the number of nodes in the branching tree.

The decision for presenting the averages for 20 instances in each table without going into detail for each set of 4 instances associated with a specific number of nodes has to do with the fact that no clear change in the results was devisable by doing so.

In terms of the lower bounds provided by the linear programming relaxation, the following conclusions can be drawn by observing tables 5-8:

- Formulations U clearly outperform formulations A. In fact, the lowest gaps for the linear relaxation bounds are always attained with formulations $U_{EK} + (36) - (24) + (33)$ and $U_{EK} + (36) - (24) + (33) + Prep$. Therefore, the linear programming relaxation of formulations U led to a better description of the convex hull of the set of feasible solutions to the CSAHLPM than the linear programming relaxation of (the corresponding) formulations A.

- The use of the preprocessing phase allows a reduction in the gap of the bound provided by the linear relaxation. Although not being a large reduction, it can be observed in the large majority of the cases.

- Another clear advantage of using the preprocessing tests is the improvement in the CPU time required to solve the linear relaxation. This was always the case with an exception in table 6.

- In formulation A_{EK} by replacing constraints (5) and (6) by (32) it was possible to obtain a model with n^2 less constraints leading to a significant reduction in the linear relaxation gap. Therefore, a better description of the convex hull of the feasible set of the CSAHLPM was obtained with that replacement.

As far as the optimal solution to the problem is concerned, by observing tables 5-8 the following conclusions can be made:

- The instances seem to become easier when the economies of scale of the set-up cost are stronger (table 5 versus table 6 and table 7 versus table 8).

- The instances seem to become easier with loose costs (table 5 versus table 7 and table 6 versus table 8).

- The results obtained show that none of the formulations clearly outperformed the others when it comes to solving the CSAHLPM optimally. Using formulations A some out-of-memory errors occurred but no time limit was reached. On the other hand, using

	Linear relaxation		Optimal solution			
Model	Average gap (%)	Average CPU (seconds)	Time limit / Out memory	Average final gap (%)	Average CPU (seconds)	Average number of nodes
A_{EK}	23.40	0.51	0/0	–	83.97	882.00
A_{EK} + Prep	23.40	0.42	0/0	–	134.08	1034.00
A_{EK} - (5) - (6) + (32)	9.69	1.02	0/0	–	114.01	1073.20
A_{EK} - (5) - (6) + (32) + Prep	9.69	0.47	0/0	–	108.48	1023.35
A_{EK} - (6) + (32) + (35)	3.68	1.81	0/1	0	182.95	2237.68
A_{EK} - (6) + (32) + (35) + Prep	3.17	1.34	0/0	–	91.42	881.15
U_{EK}	3.21	2.08	0/0	–	504.02	1613.30
U_{EK} + Prep	3.11	1.51	0/0	–	433.81	1700.45
U_{EK} + (36) - (24) + (33)	2.93	3.12	1/0	0	399.73	1029.11
U_{EK} + (36) - (24) + (33) + Prep	2.82	3.07	1/0	1.27	481.54	1080.95

Table 5: Computational results for the 20 instances of the CSAHLPM with tight costs and $\rho = 1.2$.

	Linear relaxation		Optimal solution			
Model	Average gap (%)	Average CPU (seconds)	Time limit / Out memory	Average final gap (%)	Average CPU (seconds)	Average number of nodes
A_{EK}	21.07	0.52	0/0	–	540.08	3915.40
A_{EK} + Prep	21.07	0.42	0/1	0	238.36	2323.37
A_{EK} - (5) - (6) + (32)	8.39	0.98	0/0	–	152.81	1980.75
A_{EK} - (5) - (6) + (32) + Prep	8.39	0.46	0/0	–	153.66	1989.85
A_{EK} - (6) + (32) + (35)	3.68	1.81	0/1	0	182.95	2237.68
A_{EK} - (6) + (32) + (35) + Prep	3.68	1.39	0/1	0	182.61	2253.74
U_{EK}	3.68	2.31	1/0	0	609.61	3505.50
U_{EK} + Prep	3.46	1.68	1/0	0	422.75	1760.58
U_{EK} + (36) - (24) + (33)	3.44	3.74	4/0	0	46.33	373.31
U_{EK} + (36) - (24) + (33) + Prep	3.14	4.42	3/0	0.03	396.40	633.71

Table 6: Computational results for the 20 instances of the CSAHLPM with tight costs and $\rho = 1.1$.

formulations U, no out-of-memory error occurred but in several occasions the time limit was reached. This is understandable because these formulations are heavier than formulations A in terms of the number of binary variables and the linear relaxation bound is not sharp enough to overcome the increase in the size of the formulations. This is also confirmed by the fact that the last two formulations U were the ones that in general required more CPU time (on average).

In table 9 the percentage of variables fixed in the preprocessing phase is presented. The results refer to the 4 sets of 20 instances associated with each of the tables 5-8 above. For instance, using the preprocessing phase, in the 20 instances associated with tight costs and with $\rho = 1.2$ it was possible to set (on average) 21.04% of the x-variables in models A_{EK}. The percentage of variables fixed in the preprocessing phase is quite significant if one thinks that the preprocessing tests presented in section 5 are simple deductions that can be drawn from the data of a specific instance. The simplicity of the preprocessing performed is supported by the CPU time required for it which was insignificant. In fact, for each instance the time required was lower than 0.0001 seconds. Observing table 9, one concludes that there is some tendency to fix more variables in the instances with loose costs, which may be explained by a more successful use of the preprocessing tests 3 and 7.

7 Conclusion

In this paper, an extension to the classical capacitated single-allocation hub location problem was proposed, in which the capacity of the hubs is also part of the decision process. For each potential hub it was assumed that a set of different capacities is available from which one can be chosen. Several mixed-integer linear programming formulations were proposed for the problem. A first set of formulations resulted from extending several well-known formulations for the classical problem to the new problem. Another set of formulations was motivated by the capacity choice existing for the potential hubs. A theoretical comparison was made between the models proposed aiming at finding the most promising in terms of the bound provided by the linear programming relaxation. Different sets of valid inequalities were also proposed for enhancing the models. Finally, a set of preprocessing tests was proposed with the goal of reducing the size of the formulations. The results of the computational experiments performed were presented, which considered data generated from the instances in the AP data set. These results show that for the instances analyzed, using some of the models proposed the problem can be solved efficiently by a general solver.

	Linear relaxation		Optimal solution			
Model	Average gap (%)	Average CPU (seconds)	Time limit / Out memory	Average final gap (%)	Average CPU (seconds)	Average number of nodes
A_{EK}	34.21	0.22	0/0	–	113.70	821.40
A_{EK} + Prep	34.21	0.17	0/0	–	108.07	796.65
A_{EK} - (5) - (6) + (32)	12.56	0.87	0/0	–	120.78	781.50
A_{EK} - (5) - (6) + (32) + Prep	12.55	0.42	0/0	–	113.44	761.40
A_{EK} - (6) + (32) + (35)	3.34	1.27	0/0	–	100.40	741.50
A_{EK} - (6) + (32) + (35) + Prep	3.28	0.97	0/0	–	97.96	731.10
U_{EK}	3.31	1.54	0/0	–	152.07	672.35
U_{EK} + Prep	3.03	1.29	0/0	–	156.54	664.80
U_{EK} + (36) - (24) + (33)	2.88	2.19	0/0	–	397.85	659.75
U_{EK} + (36) - (24) + (33) + Prep	2.68	1.75	0/0	–	393.52	638.52

Table 7: Computational results for the 20 instances of the CSAHLPM with loose costs and $\rho = 1.2$.

	Linear relaxation		Optimal solution			
Model	Average gap (%)	Average CPU (seconds)	Time limit / Out memory	Average final gap (%)	Average CPU (seconds)	Average number of nodes
A_{EK}	30.65	0.21	0/0	–	229.50	1432.00
A_{EK} + Prep	30.65	0.17	0/0	–	291.87	1192.45
A_{EK} - (5) - (6) + (32)	9.79	0.86	0/0	–	224.55	1556.40
A_{EK} - (5) - (6) + (32) + Prep	9.74	0.43	0/0	–	251.57	1699.40
A_{EK} - (6) + (32) + (35)	3.35	1.35	0/0	–	292.29	1655.15
A_{EK} - (6) + (32) + (35) + Prep	3.30	0.88	0/0	–	246.91	1527.60
U_{EK}	3.30	1.62	0/0	–	257.13	1128.45
U_{EK} + Prep	3.01	1.25	0/0	–	367.59	1393.05
U_{EK} + (36) - (24) + (33)	3.09	2.10	0/0	–	735.01	1098.45
U_{EK} + (36) - (24) + (33) + Prep	2.76	1.51	0/0	–	830.89	1207.25

Table 8: Computational results for the 20 instances of the CSAHLPM with loose costs and $\rho = 1.1$.

		Models A_{EK}			Models U_{EK}	
Type of costs	ρ	x_{ik}	z_k^q	y_{kl}^i	t_{ik}^q	y_{kl}^i
Tight	1.2	21.04	15.85	20.92	29.75	20.92
Tight	1.1	23.28	15.85	23.28	31.58	23.28
Loose	1.2	23.71	15.85	24.14	32.08	24.14
Loose	1.1	26.73	15.85	27.20	34.60	27.20

Table 9: Percentage of variables fixed in the preprocessing phase.

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